# 「Excel で学ぶディジタル信号処理の基礎」オンライン付録（本文中に配した演習問題の解答例）

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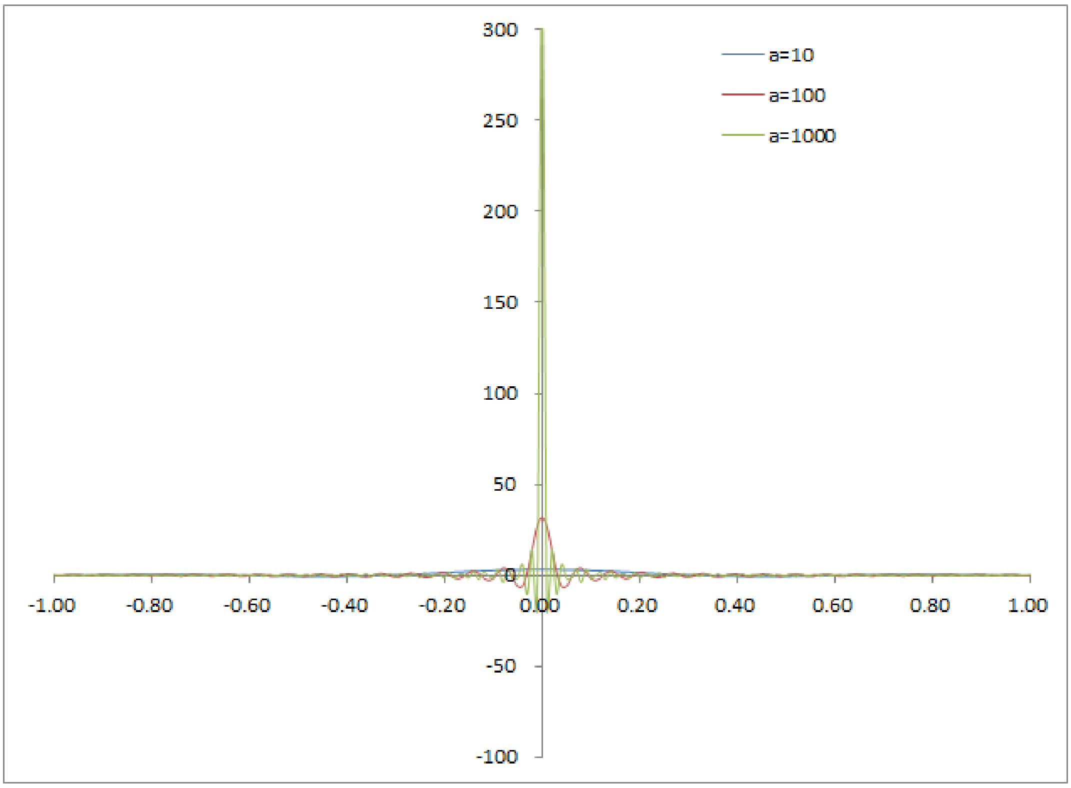
本付録の最新版・サンプルプログラム・書籍に関する情報については、コロナ社 WEB サイトの本書紹介ページ (http://www.coronasha.co.jp/np/isbn/9784339008555/) **を参照のこと。**

# 第 1 章 演習問題 略解

## 演習 1.1




## 演習 1.2

本書「付録」に示した。

## 演習 1.3

(1)

$$
\begin{aligned}
\int_0^\infty \mathrm{e}^{(a+\mathrm{j}b)t}\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t &= \int_0^\infty \mathrm{e}^{(-\mathrm{j}\omega+a+\mathrm{j}b)t}\mathrm{d}t \\
&= \left[\frac{-\mathrm{e}^{(-j\omega+a+\mathrm{j}b)t}}{\mathrm{j}\omega-(a+\mathrm{j}b)}\right]_0^\infty \\
&= \frac{1}{\mathrm{j}\omega-(a+\mathrm{j}b)} \qquad (a<0)
\end{aligned}
$$

(2)

$$\begin{aligned}\int_{-\infty}^{0} -\mathtt{e}^{(a+\mathtt{j}b)t}\mathtt{e}^{-\mathtt{j}\omega t}\mathtt{d}t &= \int_{-\infty}^{0} -\mathtt{e}^{(-\mathtt{j}\omega+a+\mathtt{j}b)t}\mathtt{d}t \\ &= \left[\frac{\mathtt{e}^{(-\mathtt{j}\omega+a+\mathtt{j}b)t}}{\mathtt{j}\omega-(a+\mathtt{j}b)}\right]_{-\infty}^{0} \\ &= \frac{1}{\mathtt{j}\omega-(a+\mathtt{j}b)}\end{aligned}$$

## **演習** 1.4

(1)

**右辺から左辺に至る変換を示す：**

$$\begin{aligned}\left(\frac{\mathtt{d}}{\mathtt{d}\omega}\right)^n X(\mathtt{j}\omega) &= \left(\frac{\mathtt{d}}{\mathtt{d}\omega}\right)^n \int_{-\infty}^{\infty} x(t)\mathtt{e}^{-\mathtt{j}\omega t}\mathtt{d}t \\ &= \int_{-\infty}^{\infty} x(t)\left(\frac{\mathtt{d}}{\mathtt{d}\omega}\right)^n \mathtt{e}^{-\mathtt{j}\omega t}\mathtt{d}t \\ &= \int_{-\infty}^{\infty} (-\mathtt{j}t)^n\, x(t)\mathtt{e}^{-\mathtt{j}\omega t}\mathtt{d}t \\ &= \mathtt{FT}\left[(-\mathtt{j}t)^n x(t)\right]\end{aligned}$$

(2)

**左辺から右辺に至る変換を示す：**

$$\begin{aligned}\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^n x(t) &= \left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^n \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mathtt{j}\omega)\mathtt{e}^{\mathtt{j}\omega t}\mathtt{d}\omega\right] \\ &= \int_{-\infty}^{\infty} (\mathtt{j}\omega)^n\, X(\mathtt{j}\omega)\mathtt{e}^{\mathtt{j}\omega t}\mathtt{d}\omega \\ &= \mathtt{FT}^{-1}\left[(\mathtt{j}\omega)^n X(\mathtt{j}\omega)\right]\end{aligned}$$

## **演習** 1.5

(1)

$\tau' = t-\tau$ **とおくと、**

$$x(t)*h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\mathtt{d}\tau$$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} x(\tau')h(t-\tau')\mathtt{d}\tau \\
&= h(t) * x(t)
\end{aligned}
$$

(2)

$\mu' = \omega - \mu$ **とおくと、**

$$
\begin{aligned}
X(\mathtt{j}\omega) * H(\mathtt{j}\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mathtt{j}[\omega-\mu])H(\mathtt{j}\mu)\mathtt{d}\mu \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mathtt{j}\mu')H(\mathtt{j}[\omega-\mu'])\mathtt{d}\mu' \\
&= H(\mathtt{j}\omega) * X(\mathtt{j}\omega)
\end{aligned}
$$

## 演習 1.6

(1)

$$
\int_{-0}^{\infty} \delta(t)\mathtt{e}^{-st}\mathtt{d}t = \mathtt{e}^{-s\cdot 0} = 1
$$

(2)

$$
\begin{aligned}
\int_{-0}^{\infty} \mathtt{e}^{(a+\mathtt{j}b)t}\mathtt{e}^{-st}\mathtt{d}t &= \int_{-0}^{\infty} \mathtt{e}^{(-s+a+\mathtt{j}b)t}\mathtt{d}t \\
&= \left[\frac{-\mathtt{e}^{(-s+a+\mathtt{j}b)t}}{s-(a+\mathtt{j}b)}\right]_{-0}^{\infty} = \frac{1}{s-(a+\mathtt{j}b)}
\end{aligned}
$$

(3)

（2）**の両辺を** $s$ **について** $m$ **回微分すると、**

$$
\int_{-0}^{\infty} (-t)^m \mathtt{e}^{(a+\mathtt{j}b)t}\mathtt{e}^{-st}\mathtt{d}t = (-1)^m \frac{m!}{(s-(a+\mathtt{j}b))^{m+1}}
$$

**よって**

$$
\int_{-0}^{\infty} t^m \mathtt{e}^{(a+\mathtt{j}b)t}\mathtt{e}^{-st} = \frac{m!}{(s-(a+\mathtt{j}b))^{m+1}}
$$

(4)

（3）**の結論の式において** $a+\mathtt{j}b=0$ **とおくと**

$$
\int_{-0}^{\infty} t^m \mathtt{e}^{-st}\mathtt{d}t = \frac{m!}{s^{m+1}}
$$

（別解）

$$
\begin{aligned}
\int_{-0}^{\infty} t^m \mathtt{e}^{-st}\mathtt{d}t &= \left[t^m \frac{\mathtt{e}^{-st}}{-s}\right]_{-0}^{\infty} - \int_{-0}^{\infty} mt^{m-1}\frac{e^{-st}}{-s}\mathtt{d}t \\
&= \int_{-0}^{\infty} \frac{mt^{m-1}}{s}\mathtt{e}^{-st}\mathtt{d}t = \cdots = \int_{-0}^{\infty} \frac{m!}{s^m}\mathtt{e}^{-st}\mathtt{d}t \\
&= \left[-\frac{m!}{s^{m+1}}\mathtt{e}^{-st}\right]_{-0}^{\infty} = \frac{m!}{s^{m+1}}
\end{aligned}
$$

(5)

$$
\begin{aligned}
\int_{-0}^{\infty} \mathtt{e}^{at}\cos(\beta t+\theta)\mathtt{e}^{-st}\mathtt{d}t &= \int_{-0}^{\infty} \mathtt{e}^{at}\frac{\mathtt{e}^{\mathtt{j}(\beta t+\theta)} + \mathtt{e}^{-\mathtt{j}(\beta t+\theta)}}{2}\mathtt{e}^{-st}\mathtt{d}t \\
&= \frac{1}{2}\left[\mathtt{e}^{\mathtt{j}\theta}\frac{1}{s-(a+\mathtt{j}\beta)} + \mathtt{e}^{-\mathtt{j}\theta}\frac{1}{s-(a-\mathtt{j}\beta)}\right] \\
&= \frac{1}{2}\cdot\frac{(s-a)(\mathtt{e}^{\mathtt{j}\theta}+\mathtt{e}^{-\mathtt{j}\theta}) + \mathtt{j}\beta(\mathtt{e}^{\mathtt{j}\theta}-\mathtt{e}^{-\mathtt{j}\theta})}{(s-a)^2+\beta^2} \\
&= \frac{(s-a)\cos\theta - \beta\sin\theta}{(s-a)^2+\beta^2}
\end{aligned}
$$

（別解）

$$
\begin{aligned}
\int_{-0}^{\infty} \mathtt{e}^{at}\cos(\beta t+\theta)\mathtt{e}^{-st}\mathtt{d}t &= \int_{-0}^{\infty} \cos(\beta t+\theta)\mathtt{e}^{-(s-a)t}\mathtt{d}t \qquad \cdots\cdots(*) \\
&= \left[\cos(\beta t+\theta)\frac{\mathtt{e}^{-(s-a)t}}{-(s-a)}\right]_{-0}^{\infty} \\
&\quad - \int_{-0}^{\infty} \beta\sin(\beta t+\theta)\frac{\mathtt{e}^{-(s-a)t}}{s-a}\mathtt{d}t \\
&= \frac{\cos\theta}{s-a} - \frac{\beta}{s-a}\int_{-0}^{\infty}\sin(\beta t+\theta)\mathtt{e}^{-(s-a)t}\mathtt{d}t \\
&= \frac{\cos\theta}{s-a} - \frac{\beta\sin\theta}{(s-a)^2}\cdot \\
&\quad \left[\left[\sin(\beta t+\theta)\frac{e^{-(s-a)t}}{-(s-a)}\right]_{-0}^{\infty} - \int_{-0}^{\infty}\beta\cos(\beta t+\theta)\frac{\mathtt{e}^{-(s-a)t}}{-(s-a)}\mathtt{d}t\right]
\end{aligned}
$$

ここで $(*)$ より

$$
\int_{-0}^{\infty} \cos(\beta t+\theta)\mathtt{e}^{-(s-a)t}\mathtt{d}t = \frac{\cos\theta}{s-a} - \frac{\beta\sin\theta}{(s-a)^2} - \frac{\beta^2}{(s-a)^2}\int_{-0}^{\infty}\cos(\beta t+\theta)\mathtt{e}^{-(s-a)t}\mathtt{d}t
$$

よって

$$
\int_{-0}^{\infty} \cos(\beta t+\theta)\mathtt{e}^{-(s-a)t}\mathtt{d}t = \frac{(s-a)\cos\theta - \beta\sin\theta}{(s-a)^2+\beta^2}
$$

## 演習 1.7

(1)

$$\begin{aligned}\mathtt{LT}\left[\sum_{m=1}^{M}\alpha_m x_m(t)\right] &= \int_{-0}^{\infty}\sum_{m=1}^{M}\alpha_m x_m(t)\mathtt{e}^{-st}\mathtt{d}t \\ &= \sum_{m=1}^{M}\alpha_m\int_{-0}^{\infty}x_m(t)\mathtt{e}^{-st}\mathtt{d}t = \sum_{m=1}^{M}\alpha_m X_m(s)\end{aligned}$$

(2)

$$\begin{aligned}\mathtt{LT}\left[\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^N x(t)\right] &= \int_{-0}^{\infty}\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^N x(t)\mathtt{e}^{-st}\mathtt{d}t \\ &= \left[\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^N x(t)e^{-st}\right]_{-0}^{\infty} - \int_{-0}^{\infty}\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^{N-1} x(t)\cdot(-s)e^{-st}\mathtt{d}t \\ &= s\int_{-0}^{\infty}\left(\frac{\mathtt{d}}{\mathtt{d}t}\right)^{N-1} x(t)e^{-st}\mathtt{d}t - x^{(N-1)}(0) \\ &= \vdots \\ &= s^N\int_{-0}^{\infty}x(t)\mathtt{e}^{-st}\mathtt{d}t - \sum_{n=0}^{N-1}\left\{s^{N-1-m}x^{(n)}(0)\right\} \\ &= s^N X(s) - \sum_{n=0}^{N-1}\left\{s^{N-1-n}x^{(n)}(0)\right\}\end{aligned}$$

(3)

$$\begin{aligned}\mathtt{LT}\left[\int_0^t x(\tau)\mathtt{d}\tau\right] &= \int_{-0}^{t}\left(\int_0^t x(\tau)\mathtt{d}\tau\right)\mathtt{e}^{-st}\mathtt{d}t \\ &= \left[\int_0^t x(\tau)\mathtt{d}\tau\frac{e^{-st}}{-s}\right]_{-0}^{\infty} - \int_{-0}^{\infty}x(t)\frac{\mathtt{e}^{-st}}{-s}\mathtt{d}t \\ &= \frac{1}{s}X(s)\end{aligned}$$

(4)

$$\mathtt{LT}\left[x(t)*h(t)\right] = \int_{-0}^{\infty}\int_{-\infty}^{\infty}x(t-\tau)h(\tau)\mathtt{d}\tau\mathtt{e}^{-st}\mathtt{d}t$$

$$
\begin{aligned}
&= \int_{-0}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\mathtt{e}^{-s(t-\tau)}h(\tau)\mathtt{e}^{-s\tau}\mathtt{d}\tau\mathtt{d}t \\
&= \int_{-\infty}^{\infty}\left[\int_{-0}^{\infty} x(t-\tau)\mathtt{e}^{-s(t-\tau)}\mathtt{d}t\right]h(\tau)\mathtt{e}^{-s\tau}\mathtt{d}\tau \\
&= X(s)\int_{-\infty}^{\infty} h(\tau)\mathtt{e}^{-st}\mathtt{d}\tau \quad (\because x(t)\ \textbf{は因果関数}) \\
&= X(s)H(s) \quad (\because h(t)\ \textbf{は因果関数})
\end{aligned}
$$

(5)

$t' = t - b$ **とおくと、**

$$
\begin{aligned}
\mathtt{LT}\left[x(t-b)\right] &= \int_{-0}^{\infty} x(t-b)\mathtt{e}^{-st}\mathtt{d}t \\
&= \mathtt{e}^{-bs}\int_{-b}^{\infty} x(t')\mathtt{e}^{-st'}\mathtt{d}t' \\
&= \mathtt{e}^{-bs}\int_{-0}^{\infty} x(t')\mathtt{e}^{-st'}\mathtt{d}t' \quad (\because x(t)\ \textbf{は因果関数}) \\
&= \mathtt{e}^{-bs}X(s)
\end{aligned}
$$

## 演習 1.8

(1.85) **式より、**

$$
\begin{aligned}
c(n) = \mathtt{FS}\left[x(t)\right] &= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\left[\cos(n\omega_0 t) - \mathtt{j}\sin(n\omega_0 t)\right]\mathtt{d}t \\
&= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\cos(n\omega_0 t)\mathtt{d}t - \frac{\mathtt{j}}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\sin(n\omega_0 t)\mathtt{d}t \\
&= \frac{1}{2}\left[a(n) - \mathtt{j}b(n)\right]
\end{aligned}
$$

(1.87) **式より、**cos **および** sin **関数が、それぞれ偶・奇関数であることに注意して** $n < 0$ **の場合を** $n \geq 0$ **の場合にまとめると、**

$$
\begin{aligned}
x(t) &= \sum_{n=-\infty}^{\infty} c(n)\left(\cos(n\omega_0 t) + \mathtt{j}\sin(n\omega_0 t)\right) \\
&= \frac{1}{2}\cdot\frac{2}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\cos(0\cdot\omega_0 t)\mathtt{d}t \\
&\quad + \sum_{n=1}^{\infty}\frac{2}{t_0}\left(\int_{-t_0/2}^{t_0/2} x(\tau)\cos(n\omega_0\tau)\mathtt{d}\tau\right)\cdot\cos(n\omega_0 t) \\
&\quad + \sum_{n=1}^{\infty}\frac{2}{t_0}\left(\int_{-t_0/2}^{t_0/2} x(\tau)\sin(n\omega_0\tau)\mathtt{d}\tau\right)\cdot\sin(n\omega_0 t)
\end{aligned}
$$

$$= \frac{1}{2}a(0) + \sum_{n=1}^{\infty} a(n)\cos(n\omega_0 t) + \sum_{n=1}^{\infty} b(n)\sin(n\omega_0 t)$$

## 演習 1.9

(1)

$$\begin{aligned}
\mathtt{FS}\left[\sum_{m=1}^{M} \alpha_m x_m(t)\right] &= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} \sum_{m=1}^{M} \alpha_m x_m(t)\mathrm{e}^{-\mathrm{j}n\omega_0 t}\mathrm{d}t \\
&= \sum_{m=1}^{M} \alpha_m \left[\frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x_m(t)\mathrm{e}^{-jn\omega_0 t}\mathrm{d}t\right] \\
&= \sum_{m=1}^{M} \alpha_m c_m(n)
\end{aligned}$$

(2)

$t' = t - b$ **とおくと、**

$$\begin{aligned}
\mathtt{FS}\left[x(t-b)\right] &= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x(t-b)\mathrm{e}^{-\mathrm{j}n\omega_0 t}\mathrm{d}t \\
&= \mathrm{e}^{-\mathrm{j}bn\omega_0}\frac{1}{t_0}\int_{-\frac{t_0}{2}-b}^{\frac{t_0}{2}-b} x(t')\mathrm{e}^{-\mathrm{j}n\omega_0 t'}\mathrm{d}t' \\
&= \mathrm{e}^{-jbn\omega_0} c_m(n) \quad \text{(}\because\ x(t)\ \textbf{は周期}\ t_0\ \textbf{の周期関数}\text{)}
\end{aligned}$$

(3)

$$\begin{aligned}
\mathtt{FS}\left[\mathrm{e}^{-j\ell\omega_0 t}x(t)\right] &= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\mathrm{e}^{-j(n-\ell)\omega_0 t}\mathrm{d}t \\
&= c(n-\ell)
\end{aligned}$$

(4)

$$\begin{aligned}
\mathtt{FS}\left[x^*(t)\right] &= \int_{-t_0/2}^{t_0/2} x^*(t)\mathrm{e}^{-jn\omega_0 t}\mathrm{d}t \\
&= \left(\frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x(t)\mathrm{e}^{-\mathrm{j}(-n)\omega_0 t}\right)^* \\
&= c^*(-n)
\end{aligned}$$

(5)

$$\begin{aligned}
\mathtt{FS}\left[x_1(t) * x_t(t)\right] &= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2}\left\{\frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x_1(\tau)x_2(t-\tau)\mathtt{d}\tau\right\}\mathtt{e}^{-\mathtt{j}n\omega_0 t}\mathtt{d}t \\
&= \frac{1}{{t_0}^2}\int_{-t_0/2}^{t_0/2}\int_{-t_0/2}^{t_0/2} x_1(\tau)\mathtt{e}^{-\mathtt{j}n\omega_0\tau}x_2(t-\tau)\mathtt{e}^{-\mathtt{j}n\omega_0(t-\tau)}\mathtt{d}\tau\mathtt{d}t \\
&= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x_1(\tau)\mathtt{e}^{-jn\omega_0\tau}\left\{\frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x_2(t-\tau)\mathtt{e}^{-jn\omega_0(t-\tau)}\mathtt{d}t\right\}\mathtt{d}\tau \\
&= \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} x_1(\tau)\mathtt{e}^{-jn\omega_0\tau}\mathtt{d}\tau c_n(n) \qquad (\because\ x_2(t)\ \text{は周期関数})
\end{aligned}$$

(6)

$$\begin{aligned}
\mathtt{FS}^{-1}\left[c_1(n) * c_2(n)\right] &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_1(m)c_2(n-m)\mathtt{e}^{-\mathtt{j}n\omega_0 t} \\
&= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_1(m)\mathtt{e}^{-jm\omega_0 t}c_2(n-m)\mathtt{e}^{-\mathtt{j}(n-m)\omega_0 t} \\
&= \sum_{m=-\infty}^{\infty} c_1(m)\mathtt{e}^{-\mathtt{j}m\omega_0 t}\sum_{n=-\infty}^{\infty} c_2(n-m)\mathtt{e}^{-\mathtt{j}(n-m)\omega_0 t} \\
&= \sum_{m=-\infty}^{\infty} c_1(m)\mathtt{e}^{-\mathtt{j}m\omega_0 t}x_2(t) \\
&= x_1(t)x_2(t)
\end{aligned}$$

## 演習 1.10

本文中に示した。

# 第 2 章 演習問題 略解

## 演習 2.1

（2.16）式の $s$ 領域に $(s-\alpha_n)$ を乗じると次式を得る：

$$(s-\alpha_n)H(s) = \frac{(s-\alpha_n)p_1}{s-\alpha_1} + \cdots + p_n + \cdots + \frac{(s-\alpha_n)p_N}{s-\alpha_N}$$

$\alpha_1, \ldots, \alpha_N$ はすべて異なるから、上式で $s \to \alpha_n$ とすると次式を得る：

$$p_n = \lim_{s \to \alpha_n} (s-\alpha_n)H(s)$$

（2.19）式の $s$ 領域に $(s-\alpha_\ell)^K$ を乗じると次式を得る：

$$\begin{aligned}(s-\alpha_\ell)^K H(s) = &\frac{(s-\alpha_\ell)^K p_1}{s-\alpha_1} + \cdots + \frac{(s-\alpha_\ell)^K p_N}{s-\alpha_N} + p_{\ell 1} \\ &+ p_{\ell 2}(s-\alpha_\ell) + \cdots + p_{\ell k}(s-\alpha_\ell)^{k-1} + \cdots + p_{\ell K}(s-\alpha_\ell)^{K-1} \cdots\cdots (*)\end{aligned}$$

$\alpha_\ell$ と $\alpha_1, \ldots, \alpha_N$ は異なるから、$(*)$ 式で $s \to \alpha_\ell$ とすると次式を得る：

$$p_{\ell 1} = \lim_{s \to \alpha_\ell} (s-\alpha_\ell)^K H(s)$$

次に $(*)$ 式を $s$ で微分すると $p_{\ell 1}$ の項は消え、$p_{\ell 2}$ は定数項、そのほかの項は $(s-\alpha_k)$ を因数に持つ。よって $s \to \alpha_\ell$ とすると次式を得る：

$$p_{\ell 2} = \lim_{s \to \alpha_\ell} \frac{\mathrm{d}}{\mathrm{d}s} \left[(s-\alpha_\ell)^K H(s)\right]$$

同様にして $(*)$ 式を $s$ で $k-1$ 回微分すると $p_{\ell 1}, \ldots, p_{\ell(k-1)}$ の項は消え、$(k-1)!p_{\ell k}$ は定数項、そのほかの項は $(s-\alpha_k)$ を因数に持つ。よって $s \to \alpha_\ell$ とすると次式を得る：

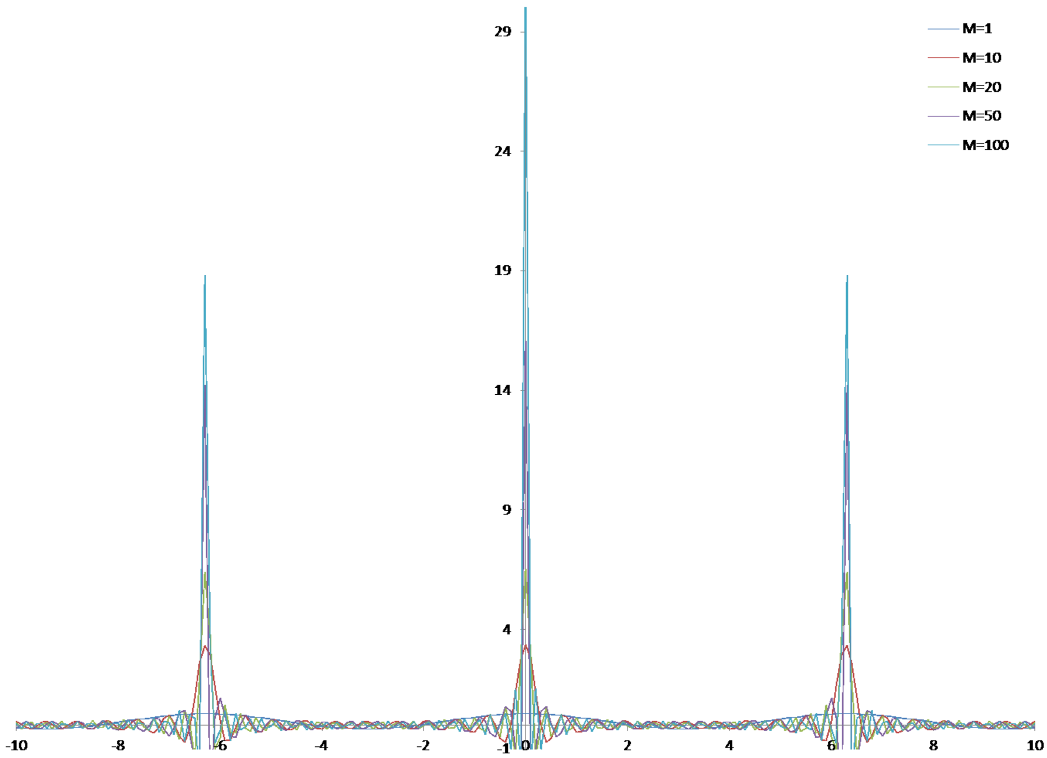
$$p_{\ell k} = \frac{1}{(k-1)!} \lim_{s \to \alpha_\ell} \left(\frac{\mathrm{d}}{\mathrm{d}s}\right)^{k-1} \left[(s-\alpha_\ell)^K H(s)\right] \qquad (k = 1, \ldots, K)$$

## 演習 2.2, 2.3, 2.4

付属 Excel ブックの ‘‘AM’’ シートを利用せよ。

# 第 3 章 演習問題 略解

## 演習 3.1




## 演習 3.2

(1)

$$\begin{aligned}
\mathtt{DTFT}\left[\sum_{m=1}^{M}\alpha_m\xi_m(k)\right] &= \sum_{k=-\infty}^{\infty}\sum_{m=1}^{M}\alpha_m\xi_m(k)\mathrm{e}^{-\mathrm{j}k\theta} \\
&= \sum_{m=1}^{M}\alpha_m\sum_{k=-\infty}^{\infty}\xi_m(k)\mathrm{e}^{-\mathrm{j}k\theta} \\
&= \sum_{m=1}^{M}\alpha_m\Theta_m(e^{\mathrm{j}\theta})
\end{aligned}$$

(2)

$$\begin{aligned}
\mathtt{DTFT}\left[\xi_m(k+n)\right] &= \sum_{k=-\infty}^{\infty} \xi_m(k+n)\mathrm{e}^{-\mathrm{j}k\theta} \\
&= \mathrm{e}^{\mathrm{j}n\theta} \sum_{k=-\infty}^{\infty} \xi_m(k+n)\mathrm{e}^{-\mathrm{j}(k+n)\theta} \\
&= \mathrm{e}^{\mathrm{j}n\theta}\Theta_m(\mathrm{e}^{\mathrm{j}\theta})
\end{aligned}$$

(3)

$$\begin{aligned}
\mathtt{DTFT}\left[\mathrm{e}^{-\mathrm{j}k\phi}\xi(k)\mathrm{e}^{-\mathrm{j}k\theta}\right] &= \sum_{k=-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}k\phi}\xi(k)\mathrm{e}^{-\mathrm{j}k\theta} \\
&= \sum_{k=-\infty}^{\infty} \xi(k)\mathrm{e}^{-\mathrm{j}k(\theta+\phi)} \\
&= \Theta(\mathrm{e}^{\mathrm{j}(\theta+\phi)})
\end{aligned}$$

(4)

$$\begin{aligned}
\mathtt{DTFT}\left[\xi^*(k)\right] &= \sum_{k=-\infty}^{\infty} \xi^*(k)\mathrm{e}^{-\mathrm{j}k\theta} \\
&= \sum_{k=-\infty}^{\infty} \xi^*(k)(\mathrm{e}^{\mathrm{j}k\theta})^* \\
&= \left(\sum_{k=-\infty}^{\infty} \xi(k)\mathrm{e}^{\mathrm{j}k\theta}\right)^* \\
&= \Theta^*(\mathrm{e}^{-\mathrm{j}\theta})
\end{aligned}$$

## 演習 3.3

(1)

$$\xi(k) * \eta(k) = \sum_{m=-\infty}^{\infty} \xi(k-m)\eta(m)$$

$$
\begin{aligned}
&= \sum_{k-m'=-\infty}^{\infty} \xi(m')\eta(k-m') \\
&= \eta(k) * \xi(k)
\end{aligned}
$$

(2)

$$
\begin{aligned}
\Theta(\mathtt{e}^{\mathtt{j}\theta}) * H(\mathtt{e}^{\mathtt{j}\theta}) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Theta(\mathtt{e}^{\mathtt{j}(\theta-\phi)})H(\mathtt{e}^{\mathtt{j}\phi})\mathtt{d}\phi \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Theta(\mathtt{e}^{\mathtt{j}\phi'})H(\mathtt{e}^{j(\theta-\phi')})\mathtt{d}\phi' \\
&= H(\mathtt{e}^{\mathtt{j}\theta}) * \Theta(\mathtt{e}^{\mathtt{j}\theta})
\end{aligned}
$$

## 演習 3.4

(1)

$$
\begin{aligned}
\mathtt{ZT}\left[\sum_{m=1}^{M} \alpha_m \xi_m(k)\right] &= \sum_{k=-\infty}^{\infty}\sum_{m=1}^{M} \alpha_m \xi_m(k) z^{-k} \\
&= \sum_{m=1}^{M} \alpha_m \sum_{m=-\infty}^{\infty} \xi_m(k) z^{-k} \\
&= \sum_{m=1}^{M} \Theta_m(z)
\end{aligned}
$$

(2)

$$
\begin{aligned}
\mathtt{ZT}\left[\xi(k+n)\right] &= \sum_{k=-\infty}^{\infty} \xi(k+n) z^{-k} \\
&= z^n \sum_{k=-\infty}^{\infty} \xi(k+n) z^{-(k+n)} \\
&= z^n \Theta(z)
\end{aligned}
$$

(3)

$$
\mathtt{ZT}\left[a^k \xi(k)\right] = \sum_{k=-\infty}^{\infty} a^k \xi(k) x^{-k}
$$

$$
\begin{aligned}
&= \sum_{k=-\infty}^{\infty} \xi(k) x^{-\frac{k}{a}} \\
&= \Theta(\frac{z}{a})
\end{aligned}
$$

(4)

$$
\begin{aligned}
\sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \xi(k-m)\eta(m) z^{-k} &= \sum_{k=-\infty}^{\infty} \xi(k-m) x^{-(k-m)} \sum_{m=-\infty}^{\infty} \eta(m) z^{-m} \\
&= \sum_{k'=-\infty}^{\infty} \xi(k') z^{-k'} \sum_{m=-\infty}^{\infty} \eta(m) z^{-m} \\
&= \Theta(z) H(z)
\end{aligned}
$$

## **演習** 3.5

(1)

$$
\begin{aligned}
\mathtt{DFT}\left[\sum_{m=1}^{M} \alpha_m f_m(k)\right] &= \sum_{k=0}^{N-1} \sum_{m=1}^{M} \alpha_m f_m(k) w_N^{-kn} \\
&= \sum_{m=0}^{M} \alpha_m \sum_{k=0}^{N-1} f_m(k) w_N^{-kn} \\
&= \sum_{m=0}^{M} \alpha_m F_m(n)
\end{aligned}
$$

(2)

$$
\begin{aligned}
\mathtt{DFT}\left[f(\mathtt{mod}[k-i, N])\right] &= \sum_{k=0}^{N-1} f(\mathtt{mod}[k-1, N]) w_N^{-kn} \\
&= w_N^{-in} \sum_{k=0}^{N-1} f(\mathtt{mod}[k-1, N]) w_N^{-(k-i)n} \\
&= w_N^{-in} F(n)
\end{aligned}
$$

(3)

$$
\mathtt{DFT}\left[w_N^{k\ell} f(k)\right] = \sum_{k=0}^{N-1} w_N^{k\ell} f(k) w_N^{-kn}
$$

$$
\begin{aligned}
&= \sum_{k=0}^{N-1} f(k) w_N^{-k(n-\ell)} \\
&= F(\mathtt{mod}[n-\ell, N])
\end{aligned}
$$

(4)

$$
\begin{aligned}
\mathtt{DFT}\left[f^*(k)\right] &= \sum_{k=0}^{N-1} f^*(k) w_N^{-kn} \\
&= \sum_{k=0}^{N-1} f^*(k) \left(w_N^{-k(N-n)}\right)^* \\
&= \left(\sum_{k=0}^{N-1} f(k) w_N^{-k(N-n)}\right)^* \\
&= F^*(N-n)
\end{aligned}
$$

(5)

$$
\begin{aligned}
\mathtt{DFT}\left[f_1(k) * f_2(k)\right] &= \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} f_1(m) f_2(\mathtt{mod}[k-m, N]) w_N^{-kn} \\
&= \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} f_1(m) w_N^{-mn} f_2(\mathtt{mod}[k-m, N]) w_N^{-(k-m)n} \\
&= F_1(n) F_2(n)
\end{aligned}
$$

# 第 4 章 演習問題 略解

## 演習 4.1, 4.2, 4.3

**付属** `Excel` **ブックの** ‘‘`FIRfilter`’’ **シートを利用せよ。**

## 演習 4.4

$$H\left(\frac{2[1-z^{-1}]}{{}_\Delta t[1+z^{-1}]}\right) = \frac{\omega_0}{\omega_0 + \frac{2(1-z^{-1})}{{}_\Delta t(1+z^{-1})}} = \frac{{}_\Delta t(1+z^{-1})}{\omega_0 + 2(1-z^{-1})}$$

$$\begin{aligned}(\omega_0 + 2(1-z^{-1})\mathcal{Z}\{\eta(k)\} &= {}_\Delta t(1+z^{-1})\mathcal{Z}\{\xi(k)\}\\ \left\{(\omega_0+2) - 2z^{-1}\right\}\mathcal{Z}\{\eta(k)\} &= {}_\Delta t(1+z^{-1})\mathcal{Z}\{\xi(k)\}\\ (\omega_0+2)\eta(k) - 2\eta(k-1) &= {}_\Delta t\left(\xi(k) + \xi(k-1)\right)\end{aligned}$$

**以上より、下記の** `IIR` **フィルタを得る：**

$$\eta(k) = \frac{2}{\omega_0+2}\eta(k-1) + \frac{{}_\Delta t}{\omega_0+2}\left(\xi(k)+\xi(k-1)\right)$$

## 演習 4.5

**付属** `Excel` **ブックの** ‘‘`Butterworth`’’ **シートを利用せよ。**

## 演習 4.6

**付属** `Excel` **ブックの** ‘‘`IIRresonator`’’ **シートを利用せよ。**

## 演習 4.7

**付属** `Excel` **ブックの** ‘‘`Singer`’’ **シートを利用せよ。**

# 第 5 章 演習問題 略解

## 演習 5.1

省略

## 演習 5.2

付属 `Excel` ブックの ‘‘WienerFilter’’ シートを利用せよ。