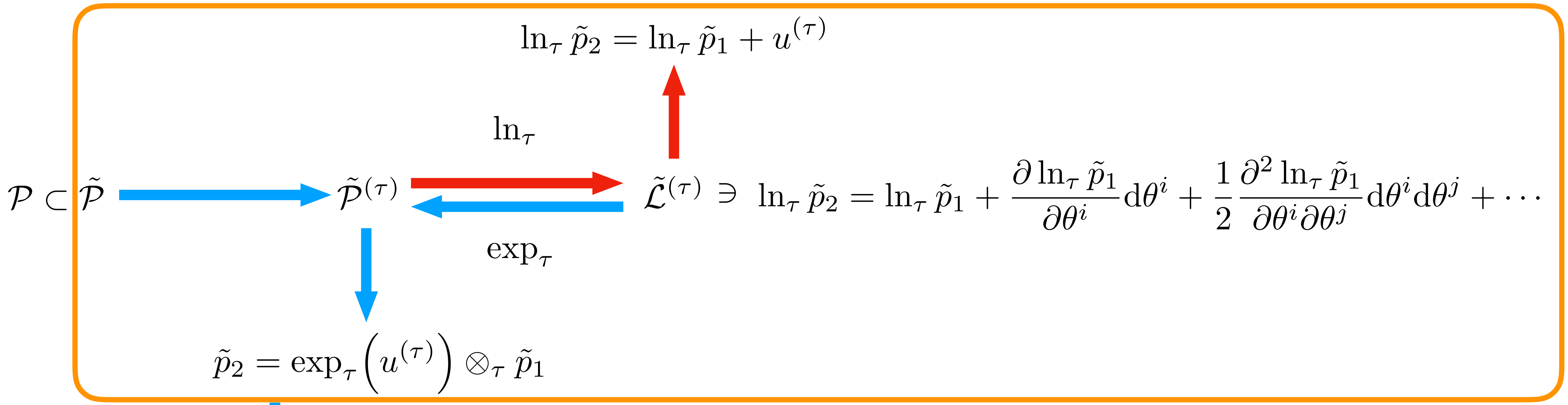
Standard Sequence



$$\tilde{p}_2 = \left\{ \tilde{p}_1^{1-\tau} + (1-\tau)\, u^{(\tau)} \right\}^{\frac{1}{1-\tau}} = \tilde{p}_1 + \tilde{p}_1^{\tau} u^{(\tau)} + F_2^{(\tau)} = \tilde{p}_1 + \tilde{p}_1^{\tau} \left(\ln_\tau \tilde{p}_2 - \ln_\tau \tilde{p}_1 \right) + F_2^{(\tau)}$$

$$F_2^{(\tau)} = \tilde{p}_2 - \tilde{p}_1 - \tilde{p}_1^{\tau} \left(\ln_\tau \tilde{p}_2 - \ln_\tau \tilde{p}_1 \right)$$

$$D^{(\tau)}(\tilde{p}_1 \| \tilde{p}_2) = \sum_{x \in \mathcal{X}} \frac{1}{\tau} F_2^{(\tau)} = \frac{1}{\tau (1-\tau)} \sum_{x \in \mathcal{X}} \left\{ \tau \tilde{p}_1 + (1-\tau)\, \tilde{p}_2 - \tilde{p}_1^{\tau} \tilde{p}_2^{1-\tau} \right\} = D^{(1-\tau)}(\tilde{p}_2 \| \tilde{p}_1)$$

$\tilde{\mathcal{P}}^{(\tau)} := \left(\tilde{\mathcal{P}}, U^{(\tau)}, \exp_\tau \right)$ $\tilde{\mathcal{P}}$ a positive cone $U^{(\tau)}$ a vector space $\mathbb{R}$ 上の N 次の多項式環

For a normalization factor,

$$C := \sum_{x \in \mathcal{X}} \exp_\tau \left(u^{(\tau)} \right) \otimes_\tau p_1$$

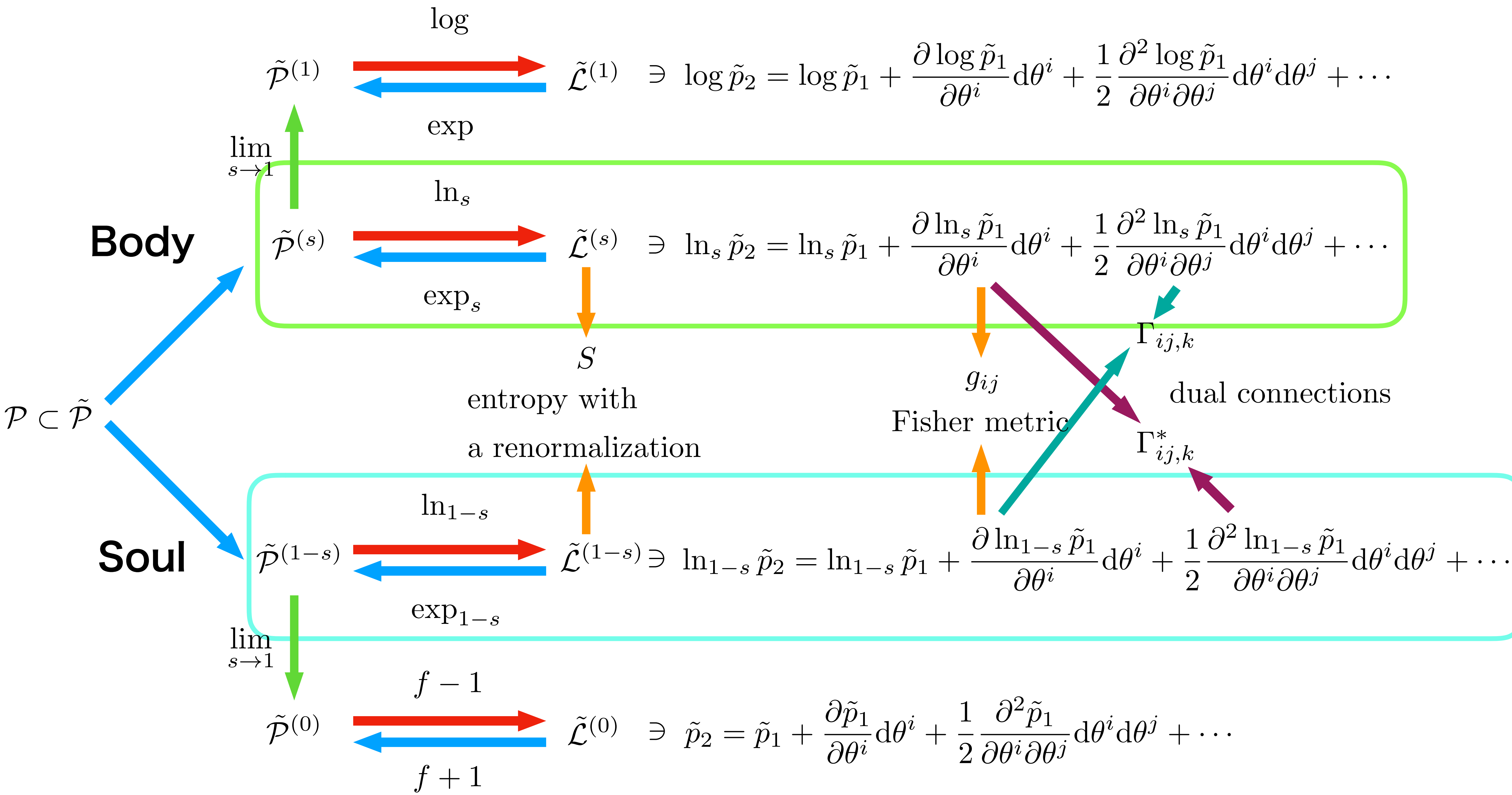
$$\ln_\tau C^{-1} = -C^{-(1-\tau)} \ln_\tau C$$

$$p_2 = \frac{1}{C} \tilde{p}_2 = \exp_\tau \left(\tilde{p}_2^{1-\tau} \ln_\tau C^{-1} \right) \otimes_\tau \tilde{p}_2 = \exp_\tau \left(\tilde{p}_2^{1-\tau} \ln_\tau C^{-1} + \ln_\tau \tilde{p}_2 \right)$$

$$= \exp_\tau \left(C^{-(1-\tau)} \left(u^{(\tau)} - \ln_\tau C + \ln_\tau \tilde{p}_1 \right) \right)$$

$(af) \otimes_\tau g$ と $f \otimes_\tau (ag)$ と $a\,(f \otimes_\tau g)$ は $a = 1$ のときを除いて互いに異なる

Dual Affine Spaces



Fisher metric の不変性を要請：２つの互いに双対なアファイン空間

$$g_{ij} = \sum_{x \in \mathcal{X}} \frac{\partial \ln_{1-s} p_1}{\partial \theta^i} \frac{\partial \ln_s p_1}{\partial \theta^j} = \sum_{x \in \mathcal{X}} \left(p_1^s \frac{\partial \log p_1}{\partial \theta^i} \right) \left(p_1^{1-s} \frac{\partial \log p_1}{\partial \theta^j} \right) = \sum_{x \in \mathcal{X}} p_1 \frac{\partial \log p_1}{\partial \theta^i} \frac{\partial \log p_1}{\partial \theta^j}$$

$$S = -\sum_{x \in \mathcal{X}} \left(\frac{1}{s} \tilde{p}^s \right) (\ln_s \tilde{p}) = -\frac{1}{s(1-s)} \sum_{x \in \mathcal{X}} (\tilde{p} - \tilde{p}^s) \qquad \tilde{p}^* = \tilde{p} \oslash_{1-s} 0 = (\tilde{p}^s + 1)^{\frac{1}{s}} \qquad \ln_{1-s} \tilde{p}^* = \frac{1}{s} \tilde{p}^s$$

Scaling coordinate

$\mathcal{P} \to \tilde{\mathcal{P}}$ (a positive cone) に座標 θ^0 を導入する

$$\mathrm{sgn}_W(\tau) = \begin{cases} +1, & (\tau = B = s) \\ -1, & (\tau = S = 1 - s) \end{cases} \qquad \tilde{p} = \exp_\tau\left(p^{1-\tau} \ln_\tau\left(\exp_s\left(\mathrm{sgn}_W(\tau)\,\theta^0\right)\right)\right) \otimes_\tau p = \exp_\tau\left(\mathrm{sgn}_W(\tau)\,\theta^0\right) \cdot p$$

$$\tilde{p}_2 = \exp_\tau\left(u^{(\tau)}\right) \otimes_\tau \tilde{p}_1 = \exp_\tau\left(u^{(\tau)} + p_1^{1-\tau} \ln_\tau\left(\exp_s\left(\mathrm{sgn}_W(\tau)\,\theta^0\right)\right)\right) \otimes_\tau p_1$$

$$\tilde{p}_2 = \exp\left(u^{(1)} + \theta^0\right) \cdot p_1 \in \tilde{\mathcal{P}}^{(1)} \underset{\exp}{\overset{\log}{\rightleftarrows}} \tilde{\mathcal{L}}^{(1)} \ni \log \tilde{p}_2 = u^{(1)} + \theta^0 + \log p_1$$

$\uparrow \lim_{s \to 1}$

Body

$$\tilde{p}_2 = \exp_s\left(u^{(s)} + p_1^{1-s}\theta^0\right) \otimes_s p_1 \in \tilde{\mathcal{P}}^{(s)} \underset{\exp_s}{\overset{\ln_s}{\rightleftarrows}} \tilde{\mathcal{L}}^{(s)} \ni \ln_s \tilde{p}_2 = u^{(s)} + p_1^{1-s}\theta^0 + \ln_s p_1$$

$\mathcal{P} \to \tilde{\mathcal{P}}$

Soul

$$\tilde{p}_2 = \exp_{1-s}\left(u^{(1-s)} + p_1^s \ln_{1-s}\left(\exp_s\left(-\theta^0\right)\right)\right) \otimes_{1-s} p_1 \in \tilde{\mathcal{P}}^{(1-s)} \underset{\exp_{1-s}}{\overset{\ln_{1-s}}{\rightleftarrows}} \tilde{\mathcal{L}}^{(1-s)} \ni \ln_{1-s} \tilde{p}_2 = u^{(1-s)} + p_1^s \ln_{1-s}\left(\exp_s\left(-\theta^0\right)\right) + \ln_{1-s} p_1$$

$\downarrow \lim_{s \to 1}$

$$\tilde{p}_2 = u^{(0)} + p_1 \exp\left(-\theta^0\right) \in \tilde{\mathcal{P}}^{(0)} \underset{f+1}{\overset{f-1}{\rightleftarrows}} \tilde{\mathcal{L}}^{(0)} \ni \tilde{p}_2 = u^{(0)} + p_1 \exp\left(-\theta^0\right)$$