英語で学ぶ
フーリエ解析とその応用

Introduction to Fourier Analysis and Its Applications

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【共著】

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ま　え　が　き

本書は大学に入学して初めてフーリエ解析を学ぶ学生の皆さんに英語でフーリエ解析とその応用を学べるように書いた書籍です。英語の専門用語には対応する日本語の用語を付記することにより学習効果や理解度を高めるようにしています。

フーリエ解析はジョゼフ・フーリエ（1768–1830）が熱伝導の法則を見出し，その現象を表現する方程式を解く方法として考え出した解析手法です。この方法には不連続な関数を含めた任意の周期関数が三角関数の無限級数で表現できることが含まれています。

フーリエはこの理論を「熱の解析的理論」という書籍にまとめて出版しました。数学者からは理論展開の厳密さについてさまざまな指摘を受けましたが，彼の提案した無限級数の収束性については後の数学者が証明しています。このことがその後の数学の発展に多大な影響を与えました。数学者が書くフーリエ解析の書籍が多い理由はここにあります。

フーリエの元の目的であった熱伝導の解析は，そののちさまざまな物質，さまざまな形状をもつ物体の熱伝導の解析に展開され実用化されています。例えば，建築設計に使われて私たちの生活空間を快適にするのに役立てたり，ロケットや飛行機，コンピュータなどさまざまな人工物で発生する熱を効果的に除去したりするために使われたりしています。

ここで視点を変えてディジタル化が進んでいる今の時代におけるフーリエ解析を考えてみましょう。この本が想定する読者にとって最も役立つ分野です。そしてその理論的基盤はフーリエ変換と逆フーリエ変換です。キーワードはいくつかあり，信号解析，スペクトル解析，高速フーリエ変換，制御工学，線形システム理論などです。これらの分野を学ぶ基礎としてフーリエ解析は必須です。

さて，工学部の多くの学生にとってフーリエ解析は難しいと思われています。新しい概念を理解するまでにたくさんの数式が並んでいるからだと思います。

しかし，基本は足し算，引き算，掛け算，割り算です。そして皆さんがなじんでいる多項式関数，三角関数，指数関数の性質をもう一度確認しましょう。そしてペンとノートを用意し，自分の頭と手で計算して例題にある周期関数をフーリエ級数に展開してみましょう。これができればフーリエ級数はすぐに理解できますし，応用することもできるようになります。

フーリエ変換も同じように学んでください。定義式を用いて例題にある関数のフーリエ変換と逆フーリエ変換を計算し，それを通して複雑な数式に慣れてください。そうすることで本書の後半まで臆することなく読み進めていく解析力が身につくと思います。

本書を手にされた皆さんが本書を読破し，さらなる高みを目指して学んでいくことを切に願っています。

最後に，日本大学生産工学部機械工学科の綱島均 教授に鉄道状況診断データの使用許可をいただき，また，株式会社 CrowLab および宇都宮大学バイオサイエンス教育研究センターの塚原直樹 博士にカラスの鳴き声データをわざわざ拙作のために作成していただき，この場を借りて感謝いたします。なお，例題の作成に協力した東京工科大学大学院の劉鉄城 氏と，原稿を詳しくチェックし貴重な助言をくださった中国湖南工業大学の何静 氏，および中国地質大学（武漢）の梅啓程 氏，羅望 氏，孫一仆 氏，周宇健 氏と賀文朋 氏にお礼を申し上げます。また，出版にあたり心暖かく見守ってくださったコロナ社に深く感謝いたします。

2023 年 3 月

佘　錦華，宮本　皓，川田　誠一

Preface

This book is written for students who learn Fourier analysis for the first time after entering university so that they can learn Fourier analysis and its applications in English. To enhance the learning effectiveness and comprehension of students studying at universities in Japan, we add the corresponding Japanese terms to English technical terms.

Fourier analysis is an analysis technique devised by Joseph Fourier (1768–1830) as a method of finding the law of heat conduction and solving the equation that describes physical phenomena. This method includes the fact that any periodic function, including discontinuous functions, can be represented by an infinite series of trigonometric functions.

Fourier published his theory in a book entitled *The Analytic Theory of Heat*. Mathematicians pointed out the rigor of theory development. Later, mathematicians proved the convergence of the infinite series he proposed. This had an enormous impact on the subsequent development of mathematics. This is why there are a huge number of books on Fourier analysis written by mathematicians.

The analysis of heat conduction, which was Fourier's original purpose, was later developed into the analysis of heat conduction for various substances and with various shapes and put into practical use. For example, it is used in architectural design to make our living spaces comfortable and is used to effectively remove the heat generated by various man-made objects such as rockets, airplanes, and computers.

Let us change our perspective and consider Fourier analysis in this age of digitalization. This is the area where Fourier analysis is the most useful field for the readers of this book. The theoretical basis is Fourier and inverse

Fourier transforms. Take signal analysis, spectrum analysis, fast Fourier transform, control engineering, linear system theory, and other keywords into consideration. Fourier analysis is essential as a basis for studying these fields.

Fourier analysis is considered to be a difficult subject for engineering students because there are many formulas involved in the process of understanding a new concept. Nevertheless, the basics are addition, subtraction, multiplication, and division. It is also important to reconfirm the properties of polynomial functions, trigonometric functions, and exponential functions that we are all familiar with. It is strongly recommended to prepare a pen and a notebook, and expand periodic functions to a Fourier series in examples using your head and hands. If you can do this, you will be able to quickly understand the Fourier series and apply it.

Learn the Fourier transform in the same way. Computing Fourier and inverse Fourier transforms of functions examples from definitions in this book ensures getting used to complex formulas. By doing so, a reader will acquire analytical skills to read the latter half of this book without hesitation.

We sincerely hope that everyone who picks up this book will read through it and learn to take it to a higher level.

Finally, we would like to take this opportunity to thank Professor Hitoshi Tsunashima of the Department of Mechanical Engineering, College of Industrial Technology, Nihon University for permission to use the railroad diagnostic data, and Dr. Naoki Tsukahara of CrowLab Inc. and the Center for Bioscience Education and Research and Education, Utsunomiya University for creating crow-call data for this book. We would like to express heartfelt appreciation to Mr. Tiecheng Liu of the Graduate School of Tokyo University of Technology for his help in preparing examples, to Prof. Jing

He of Hunan University of Technology, Zhuzhou, China, and Mr. Qicheng Mei, Mr. Wang Luo, Mr. Yipu Sun, Mr. Yujian Zhou, and Mr. Wenpeng He of China University of Geosciences, Wuhan, China for their careful checking of the manuscript and for her valuable advice. We are deeply grateful to Corona Publishing Co. Ltd. for warmly watching over the publication.

March 2023

Jinhua She, Kou Miyamoto, Seiichi Kawata

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1 Overview of Fourier Analysis

Nowadays, *computed tomography* (CT, コンピュータ断層撮影) scans play an important role in the medical field. It provides us with visual information about the inside of a body to make it easy to diagnose diseases. This requires a technology called the *Fourier transform* (フーリエ変換) to scan a human body with radiation and construct internal images of the body.

Just like listening to music and writing it down in pitch strengths, the Fourier transform of an original function clearly reveals its characteristics in a special domain [it is called the *frequency domain* (周波数領域)] that cannot be seen in the *time domain* (時間領域). Since many physical and engineering phenomena can easily be analyzed using the Fourier transform, as a mathematical tool of applied analysis, it is of† central importance in signal processing and system analysis in physical science, applied mathematics, and engineering.

1.1 History of Fourier Analysis

Jean-Baptiste Joseph Fourier (March 21, 1768–May 16, 1830) was a French mathematician and physicist. He showed that heat conduction in solid bodies could be analyzed in terms of an infinite mathematical series, the Fourier series, in *Théorie analytique de la chaleur* (*The Analytical Theory of Heat*) in 1822. This is the beginning of the *Fourier analysis* (フーリエ解析).

The basic idea is that a periodic function can be represented by a *Fourier*

† be of ～：～という特徴をもつ（= have）。

series (フーリエ級数) (a superposition of simple sinusoidal waves) and an *aperiodic function* (非周期関数) can be represented by a *Fourier integral* (フーリエ積分). If a function depends on time (for example, a sound) or space (for example, a picture), then we decompose it into a function in *temporal frequency* (時間周波数) or *spatial frequency* (空間周波数) to extract its properties. This is called a Fourier transform, which is the cornerstone of the modern digital age. Almost all of the signals around us can be transformed into processable signals using the Fourier analysis.

In science and engineering, it is important to mathematically express and analyze phenomena and derive methods to clarify their essence. The Fourier analysis provides us with a tool for such a purpose. The analysis and its philosophy have an essential influence on science and engineering. Familiarity with this analysis will be of great help in learning specialized subjects.

1.2 Illustrative Examples

This section presents three examples among others† to show how the Fourier analysis is used to solve problems.

1.2.1 Shape of Sound

There are two types of sounds: a pure tone and a compound sound. A pure tone is a sound with a sinusoidal waveform, or in other words, a sine wave of any frequency and amplitude. The sound we usually hear is a compound sound, which is a sound composed of several sinusoidal waveforms superimposed on the main one. In **Figure 1.1**, a compound sound is decomposed into three pure tones.

† among others：数ある～の中で。

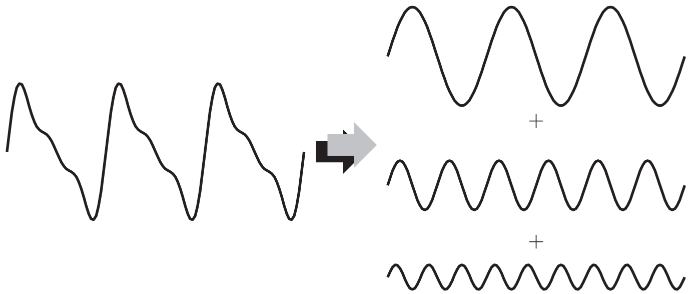

Figure 1.1 Decomposition of a compound sound into three pure tones

Examining the frequency components of pure tones in a compound sound lays the foundation for speech analysis and is widely used in speech recognition, speech synthesis, medical studies including vocal loading, audio forensics, and so on.

1.2.2 Image Processing

While a sound is a function of time, an image is a function of space. Thus, an image is a two-dimensional (height and width) function. In the previous example, the frequency of a sound is the number of occurrences of a repeating event per unit of time because a sound is a function of time. In contrast[†1], if we define a spatial frequency to be the number of striped patterns per unit length, we can analyze images in the same way as dealing with sounds.

An example is shown in **Figure 1.2**(b), in which the central part shows low-frequency components; and the outer part, high-frequency ones[†2]. Observing an image in the frequency domain provides us with a quite different viewpoint. This makes it easy to extract the nature hidden behind complex changes.

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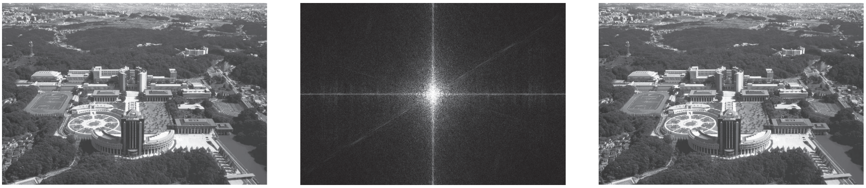
†1 in contrast：これに対して，それに対して。
†2 “,” の使用で同じ動詞の繰り返しを避ける：このコンマは “shows” を意味する。



(a) Original image
(b) 2-dimensional Fourier transformation of (a) (Low-frequency components at center)
(c) 2-dimensional inverse Fourier transformation of (b)

Figure 1.2 Fourier transform of a two-dimensional image

1.2.3 Health Monitoring of Railroad Tracks

Track safety monitoring and management are important. Since damage such as rail breakage may lead to serious accidents such as derailment, it is necessary to detect signs of cracks and other damage at an early stage before rail damage occurs to prevent accidents. Equipping an ordinary train with some simple sensors makes it possible to diagnose the condition of tracks while the train is in commercial operation.

Train tracks gradually change due to the repeated passage of trains and natural phenomena. The shape of the rail, which is the running road surface of a train, changes in the longitudinal direction. This is called track irregularity. Rail corrugation is a phenomenon in which the top of the railhead wears from several centimeters to a dozen centimeters. Large rail corrugation causes loud noise and vibrations, and damages to rail material. Note that track irregularity causes big vibrations in low frequencies; and rail corrugation, in high frequencies. It is possible to detect track conditions from noise in a train. **Figure 1.3** [16),†] shows a big peak observed at a low frequency caused by rail corrugation.

† 肩付き数字は巻末の文献番号を示す。

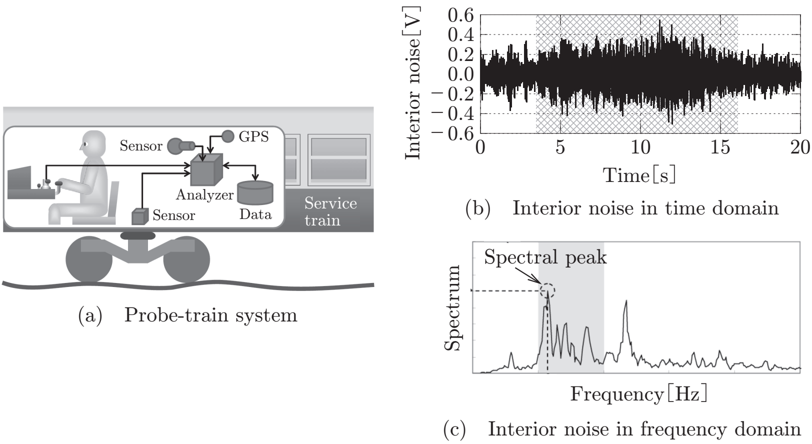


Figure 1.3 Probe-train system and monitoring results

1.2.4 Structural Control

The climate and topography of Japan make it particularly vulnerable to natural disasters. Japan is located in the Ring of Fire (also known as the Circum-Pacific Mobile Belt) where seismic and volcanic activities occur constantly. Although the country covers only 0.25% of the land area on the planet, 18.5% of earthquakes in the world occur in Japan, which is an extremely high number.

About 30 typhoons originate over the Northwest Pacific Ocean every year. Okinawa lies right in the heart of Typhoon Alley. Seven or eight typhoons every year pass over Okinawa, and about three hit the Japanese main islands, especially Kyushu and Shikoku. They have hurricane-strength winds, sometimes up to 300 km/h.

How to protect structures from earthquakes, typhoons, and other types of disasters is a big issue. Many advanced theorems and technologies have been applied to deal with them.

Now, we take an earthquake as an example. Different seismic waves

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英語で学ぶ　**フーリエ解析とその応用**
Introduction to Fourier Analysis and Its Applications

2023 年 5 月 11 日　初版第 1 刷発行　★

検印省略

著 者	佘 錦 華
	宮 本 皓
	川 田 誠 一
発 行 者	株式会社 コ ロ ナ 社
	代 表 者 牛 来 真 也
印 刷 所	三 美 印 刷 株 式 会 社
製 本 所	有限会社 愛 千 製 本 所

112–0011　東京都文京区千石 4–46–10
発 行 所　株式会社　**コ ロ ナ 社**
CORONA PUBLISHING CO., LTD.
Tokyo Japan
振替 00140–8–14844・電話(03)3941–3131(代)
ホームページ https://www.coronasha.co.jp

ISBN 978–4–339–06127–7　C3041　Printed in Japan　（新宅）